Comment on "Evidence of Non-Mean-Field-Like Low-Temperature Behavior in the Edwards-Anderson Spin-Glass Model"

Reference [1] compares the low-temperature phase of the 3D Edwards-Anderson model (EA) to the Sherrington-Kirkpatrick model (SK), studying the overlap distributions $P_{\mathcal{J}}(q)$ and concluding that the two models behave differently. A similar analysis using state-of-the-art, larger data sets for EA (generated with Janus [2] in [3]) and for SK (from [4]) leads to a very clear interpretation of the results of [1], showing that EA behaves as predicted by the replica symmetry breaking (RSB) theory.

Reference [1] studies $\Delta(\kappa, q_0)$, probability of finding in $P_{\mathcal{J}}(q)$ a peak greater than κ for $q < q_0$. In a RSB system, $\lim_{N\to\infty}\Delta(\kappa, q_0) = 1$. Figure 5 of [1] shows that, at fixed q_0 and at the same T/T_c, Δ grows for SK but seems to reach a plateau for EA. In the inset of Fig. 1 we show that, considering larger systems ($N \le 32^3$ as opposed to $N \le 12^3$ of [1]), Δ clearly grows with N also for EA. We use the same value of q_0 as in [1] and $T = 0.703$. Even this simple analysis is sufficient, when using state-of-the-art lattice sizes, to show that Δ has the same qualitative behavior in both models.

Still, the choice of comparing data for different models at the same T/T_c and N does not have a strong basis. Indeed, according to the mean-field picture, the fluctuations of the $P_{\mathcal{J}}(q)$ are ruled by the shape of the averaged $P(q)$ [5], so it is more appropriate to select T such that $P(q)$ is similar for EA and SK. Now, it is universally accepted that the peak at $q = q_{\mathrm{EA}}$ in $P(q)$ grows with N more slowly for EA, so the simplest assumption that all the individual peaks for $q < q_{\mathrm{EA}}$ scale at the same rate would already explain the results reported in [1].

According to RSB theory, in the large-N limit, $P_{\mathcal{J}}(q) = \sum_\gamma W_\gamma \delta(q - q_\gamma)$. Let us assume that for large but finite N, the weight distribution is unchanged, but the delta functions are smoothed to a finite height $H(N)$ [6]. The self-averaging peak at $q = q_{\mathrm{EA}}$ will also be smoothed, so we can estimate $H(N) \sim P(q_{\mathrm{EA}})$. $\Delta(\kappa, q_0)$ is the probability of finding a peak with weight $W_\gamma > \kappa/H(N)$, which, for small q_0, is $\Delta(\kappa, q_0) \sim [\kappa/H(N)]^{-I(q_0)} \sim [P(q_{\mathrm{EA}})/\kappa]^{I(q_0)}$, where $I(q_0) = \mathbb{P}(|q| < q_0)$ [5].

We show Δ at $T = 0.4$ for SK (top) and at $T = 0.703$ for EA (middle), where the temperatures are such that $P(0)$ are very similar (for the largest systems, q_0 ranges from 0.02 to 0.44). The curves show universal scaling for large N. The bottom panel compares Δ for SK and EA using similar effective sizes.

In short, the simple assumption that peaks for all values of q scale at the same rate is consistent with the numerical data and explains the slower growth of Δ with N for EA. Therefore, contrary to the claims in [1], we find no quantitative difference between EA and SK, as long as one is careful when comparing nonuniversal quantities and uses state-of-the-art system sizes.

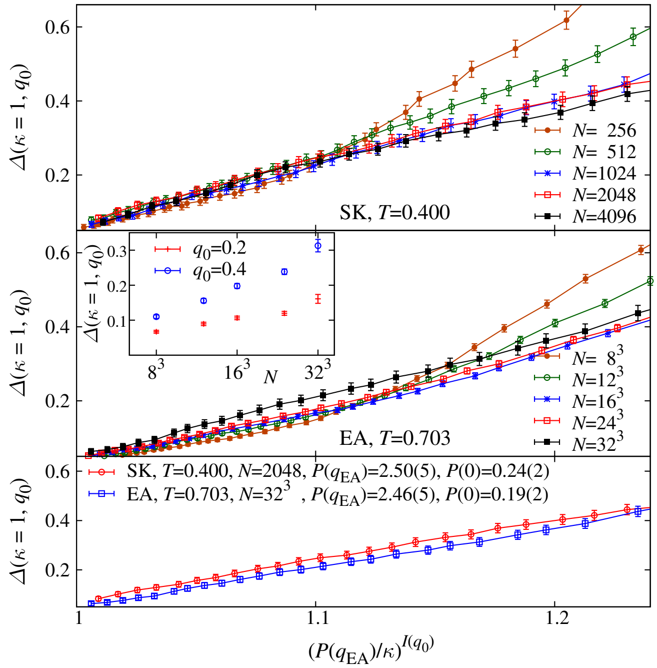

FIG. 1 (color online). $\Delta(\kappa, q_0)$ against $[P(q_{\mathrm{EA}})/\kappa]^{I(q_0)}$ for SK (top) and EA (middle). Inset: $\Delta(\kappa, q_0)$ for fixed q_0 for the EA model. Bottom: comparison of the EA and SK models for similar values of $P(q_{\mathrm{EA}})$.

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